Quantum sensors:
taming the two-stage architecture

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Quantum Optics X, Toruń, 09.09.2021

Fundusze Europejskie Inteligentny Rozwój
Rzeczpospolita Polska
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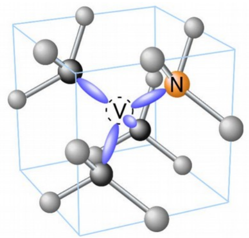
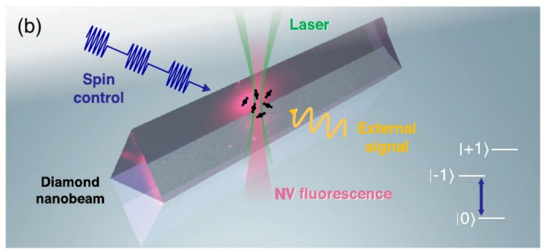
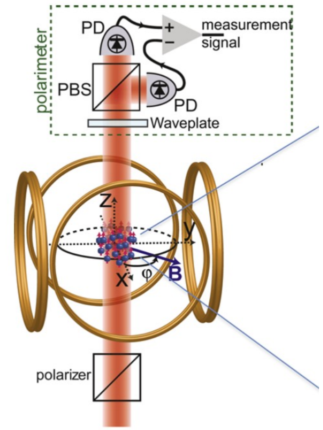
Quantum sensors

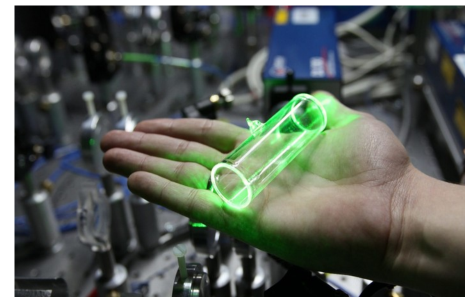
Solid-state (e.g. NV-centres in diamond)



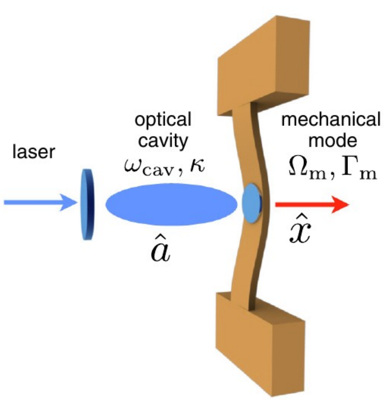


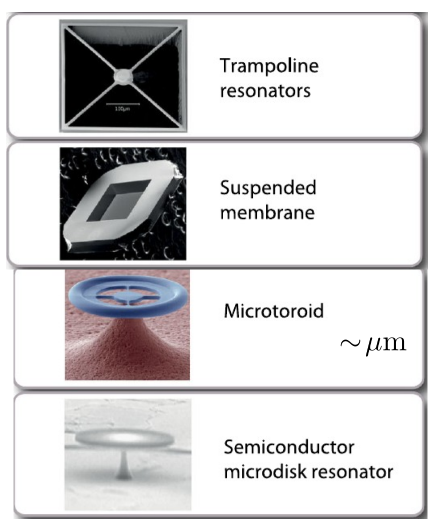
Atomic sensors



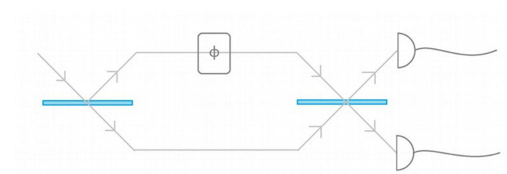


Optomechanical sensors

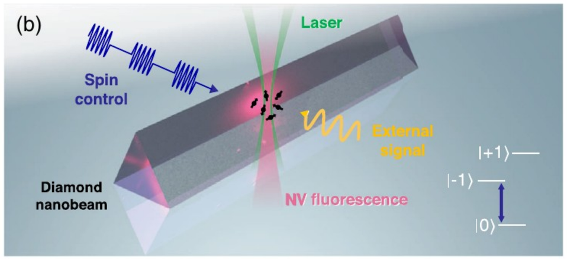


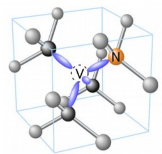


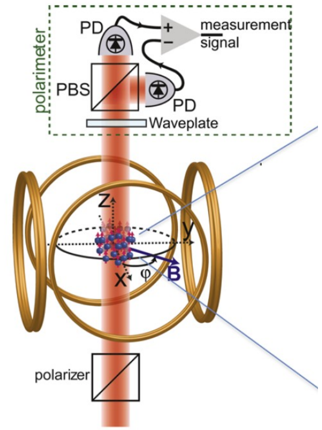
Optical interferometers

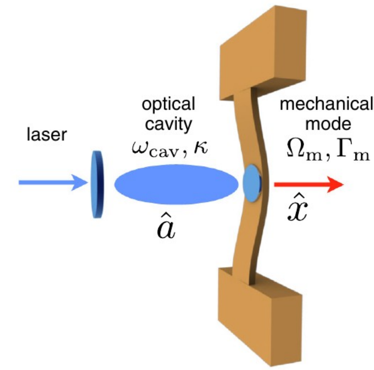


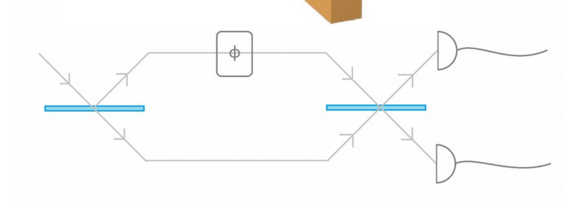
Quantum sensors: the 'two-stage' architecture







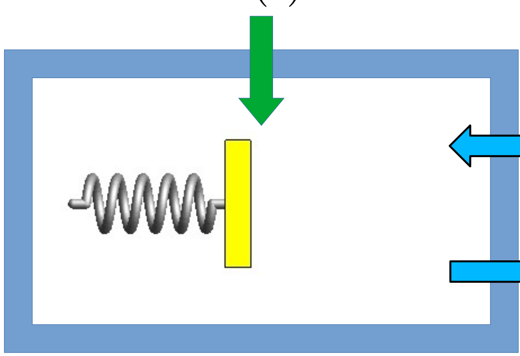




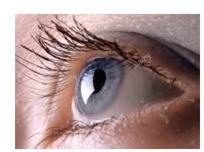
Schematic operation as a ***transducer***

sensed quantity (field, force, temperature,...)

$\theta(t)$



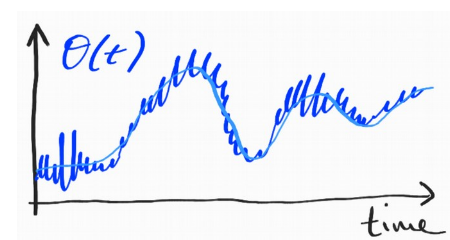
signal in



signal out

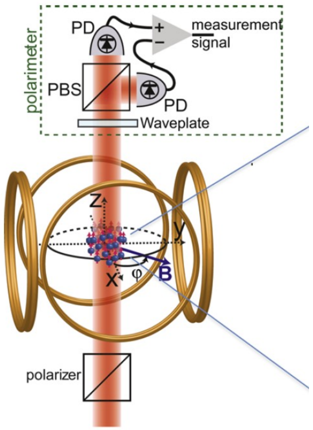
$y(t)$

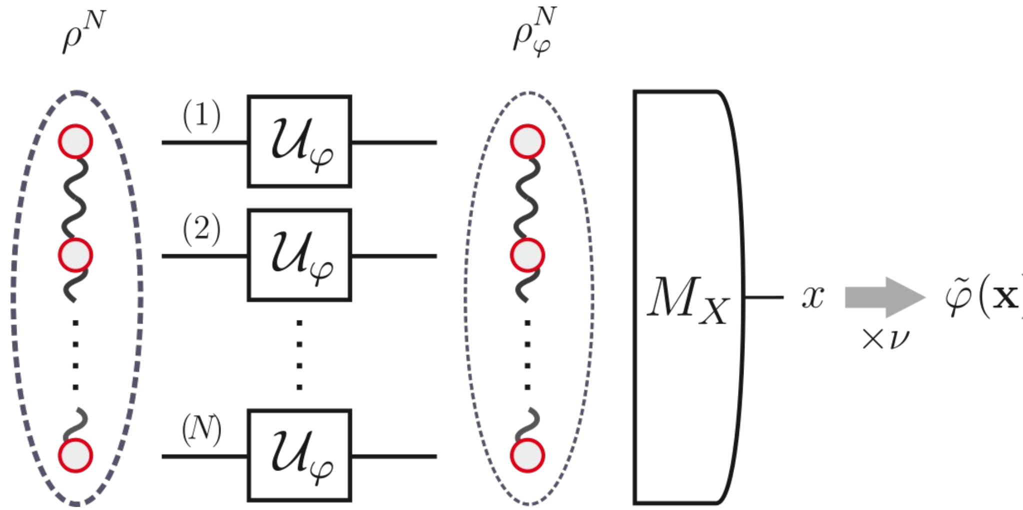
well-isolated quantum system



Measurement forms an **integral part** of sensor dynamics operating in **real time**!

Quantum metrology: (too) theoretical approach





<u>Aim.</u> Minimise the mean-squared error: $\Delta^2\tilde{\varphi} = \langle(\tilde{\varphi}-\varphi)^2\rangle$

Ultimate bound on <u>sensitivity</u> to small fluctuations of the parameter (in the $\nu \to \infty$ limit):

'tour de force'

Quantum Cramer-Rao Bound

$$\nu\,\Delta^2\tilde{\varphi} \underset{(\nu\to\infty)}{\geq} \frac{1}{F_Q[\rho_\varphi^N]}$$

Quantum Fisher Information (QFI)

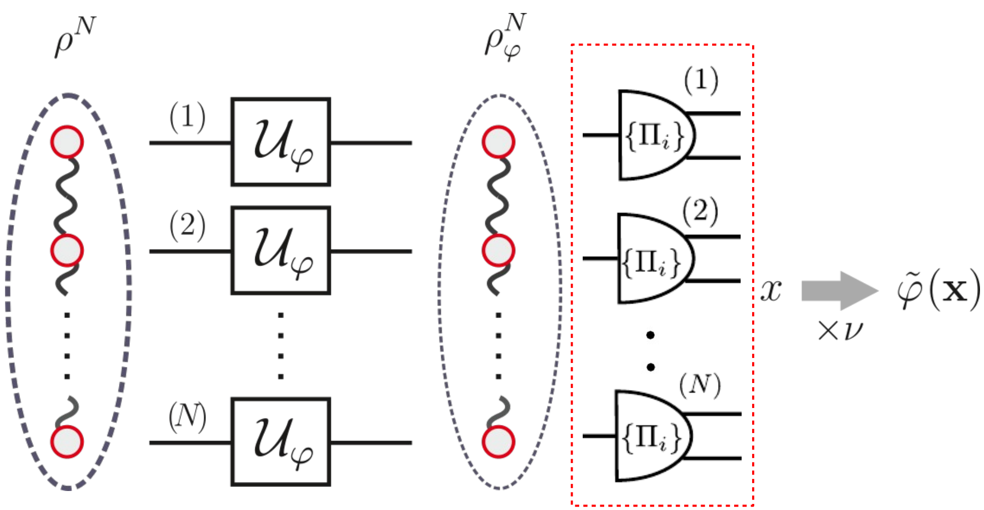
$$F_Q[\rho_\varphi^N] := \mathrm{Tr}\{\rho_\varphi^N\, L^2\}$$

$$\frac{d\rho_\varphi^N}{d\varphi} = \frac{1}{2}(L\,\rho_\varphi^N + \rho_\varphi^N\, L)$$

symmetric logarithmic derivative

- ***Local (frequentist) estimation*** for *unbiased estimators* and *asymptotic statistics*
  —in contrast to the *Bayesian approach*.
- ***Optimised over all measurements***/inference strategies (for fixed measurement/probabilities **classical FI**).

Quantum metrology: (too) theoretical approach



Unitary local encoding: $\mathcal{U}_\varphi = \mathrm{e}^{-\mathrm{i}\varphi \hat{H}}$

$$\nu\,\Delta^2\tilde{\varphi} \geq \frac{1}{F_{\mathrm{Q}}[\rho^N]}$$

$$F_{\mathrm{Q}}[\rho^N] := 2\sum_{k,l}\frac{(\lambda_k-\lambda_l)^2}{\lambda_k+\lambda_l}|\langle\psi_k|\hat{H}|\psi_l\rangle|^2$$

$$F_{\mathrm{Q}}\left[\psi^N\right] := 4\;\Delta^2\hat{H}\Big|_{\psi^N}$$

$$\hat{H} = \frac{1}{2}\sum_n \hat{h}^{(n)} \text{ with } \hat{h} = \frac{1}{2}\hat{\sigma}_z$$

QFI is *additive* and *convex*:

$$F_{\mathrm{Q}}[\otimes_i \varrho_i] = \sum_i F_{\mathrm{Q}}[\varrho_i]\,,\quad F_{\mathrm{Q}}\left[\sum_i p_i\varrho_i\right] \leq \sum_i p_i F_{\mathrm{Q}}[\varrho_i]$$

For *separable* and *entangled* states, we have:

$$\Delta^2\hat{H}_{\mathrm{sep}} \leq \left(s(\hat{h})^2/4\right)N \qquad \Delta^2\hat{H}_{\mathrm{ent}} \leq \left(s(\hat{h})^2/4\right)N^2$$

⇩

For ***separable states*** precision is bounded by the **Standard Quantum Limit**:

$$F_{\mathrm{Q}}\left[\rho^N_{\mathrm{sep}}\right] \leq N \quad\Longrightarrow\quad \boxed{\Delta^2\tilde{\varphi} \geq \frac{1}{N}}$$ **SQL**

For ***entangles states*** pricision is bounded by the **Heisenberg Limit**:

$$F_{\mathrm{Q}}\left[\psi^N_{\mathrm{GHZ}}\right] = N^2 \quad\Longrightarrow\quad \boxed{\Delta^2\tilde{\varphi} \geq \frac{1}{N^2}}$$ **HL**

GHZ (or N00N) state: $|\psi^N_{\mathrm{GHZ}}\rangle := \frac{1}{\sqrt{2}}\left(|0^N\rangle + |1^N\rangle\right)$

Local measurements are sufficient!

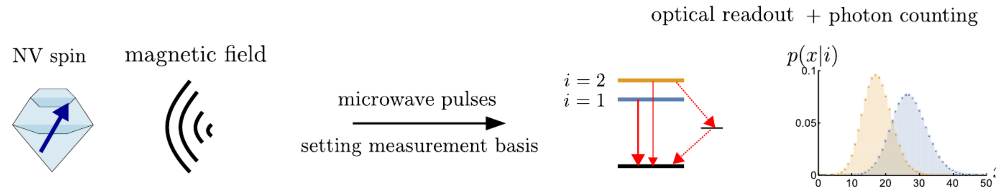
Quantum sensors: three challenges

A) **Measurements are never perfect.**

"Quantum metrology with imperfect measurements"

Yink Loong Len → Poster Session B [**arXiv:2109.01160**]

(general) NV-centres

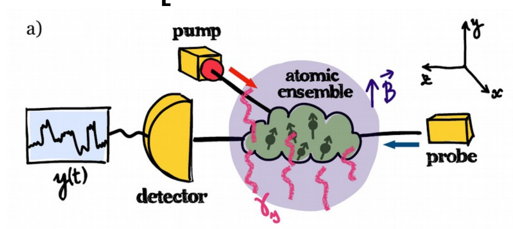


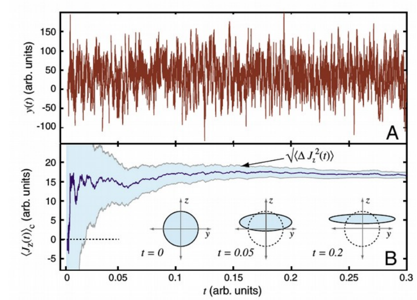
B) Sensing is performed **continuously in time**.

"Noisy atomic magnetometry in real time"

Julia Amoros Binefa → Poster Session A [**arXiv:2103.12025**]

atomic sensors



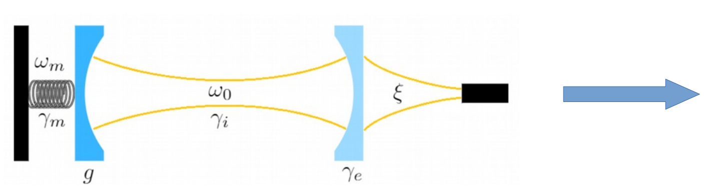


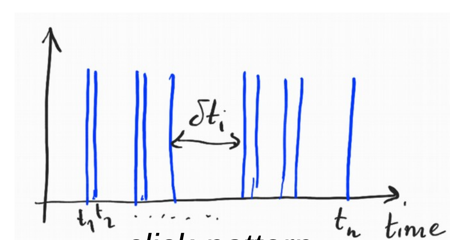
C) **Efficient statistical inference** from the recorded data is crucial.

"Enhancing performance of optomechanical sensors by continuous photon-counting"

Lewis Clark & Bartosz Markowicz → Poster Sessions A&B

optomechanical sensors





click-pattern

Quantum sensors: three challenges

A) **Measurements are never perfect.**

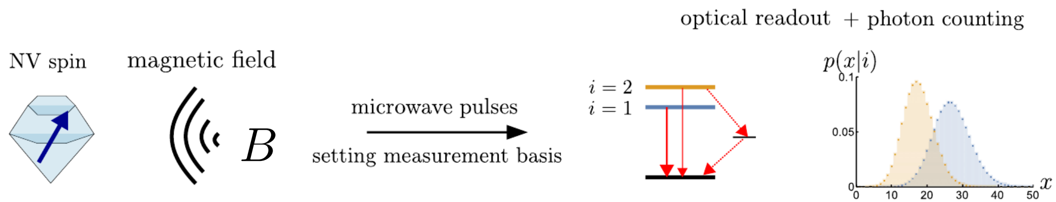
"*Quantum metrology with imperfect measurements*" [**arXiv:2109.0116]**

Yink Loong Len → Poster Session B

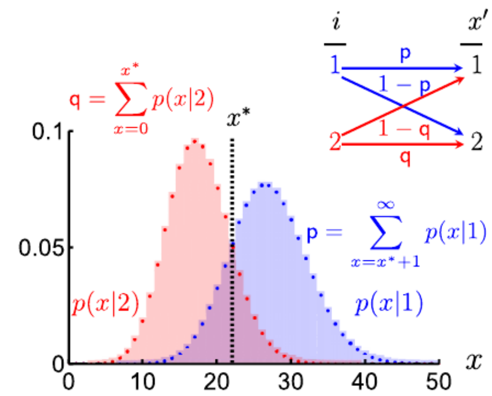
[arXiv:2109.01160]

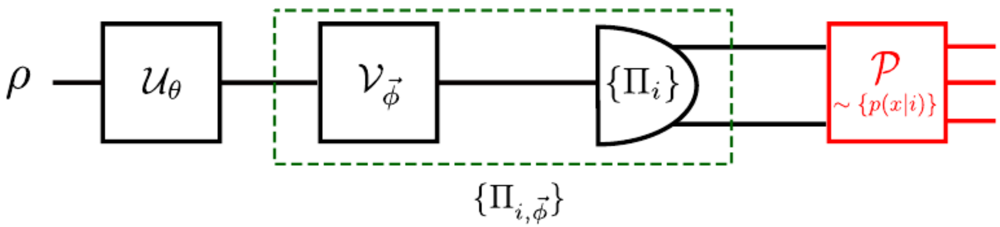
Imperfect measurements

A <u>single</u> NV sensing a constant magnetic field *B*:



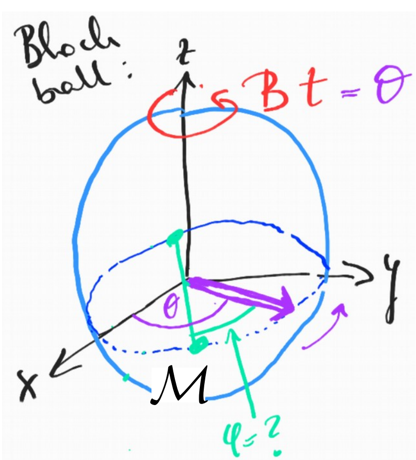
repetitive readout (QND) procedure

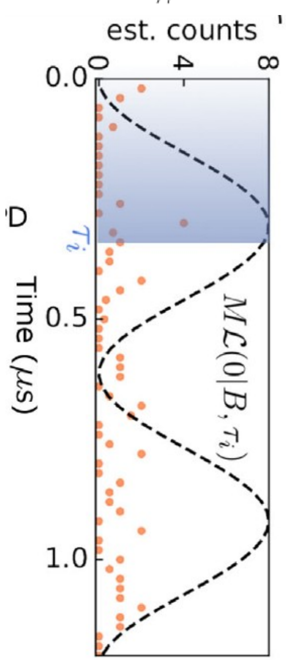


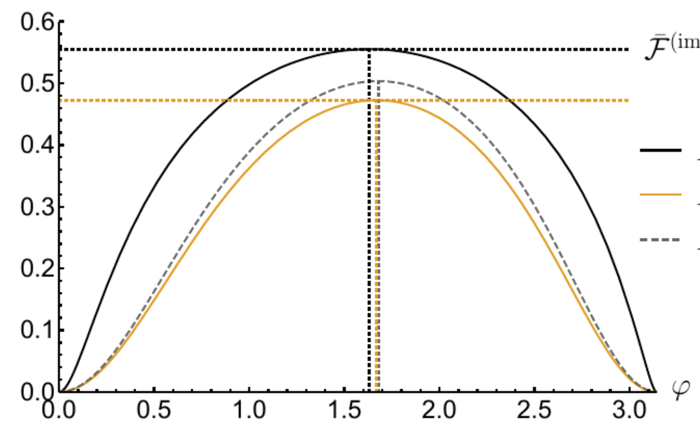


threshold method → **bit-flip errors**

What is then the optimal basis to measure the NV-centre?





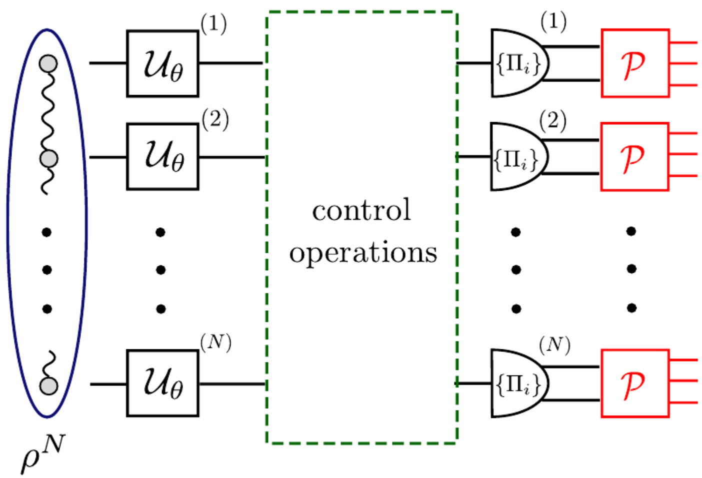


Theorem (general):

$$\mathcal{F}^{(\text{im})} = \gamma_{\mathcal{M}}\, \mathcal{F}[\psi(\theta)]$$

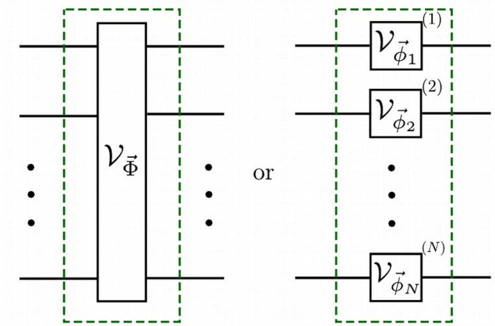
Imperfect measurements

Sensing with <u>multiple N probes</u> and noisy readout ($\mathcal{P}$):



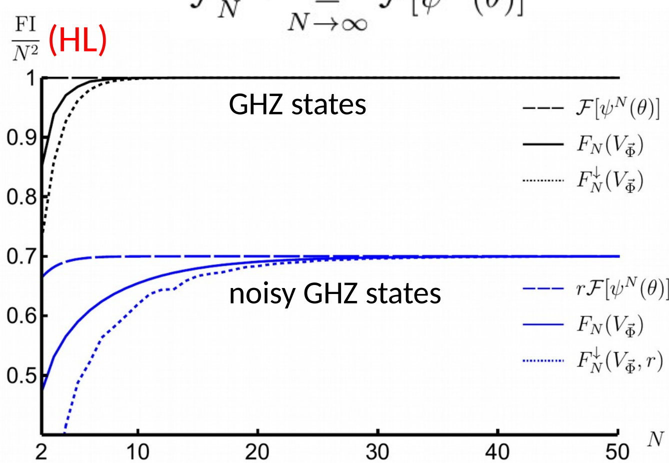
What is the optimal sensitivity you can still attain? HL?

The answer strongly depends on what **control operations** you have.



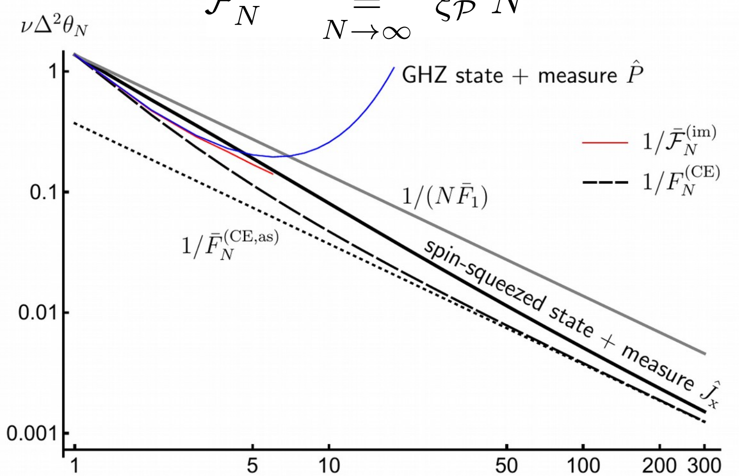
<u>**Go theorem** for *global* ops</u>

$$\mathcal{F}_N^{(\mathrm{im})} \underset{N\to\infty}{=} \mathcal{F}[\psi^N(\theta)]$$



<u>**No-go theorem** for *local* ops</u>

$$\mathcal{F}_N^{(\mathrm{im})} \underset{N\to\infty}{=} \xi_{\mathcal{P}}\, N$$



Quantum sensors: three challenges

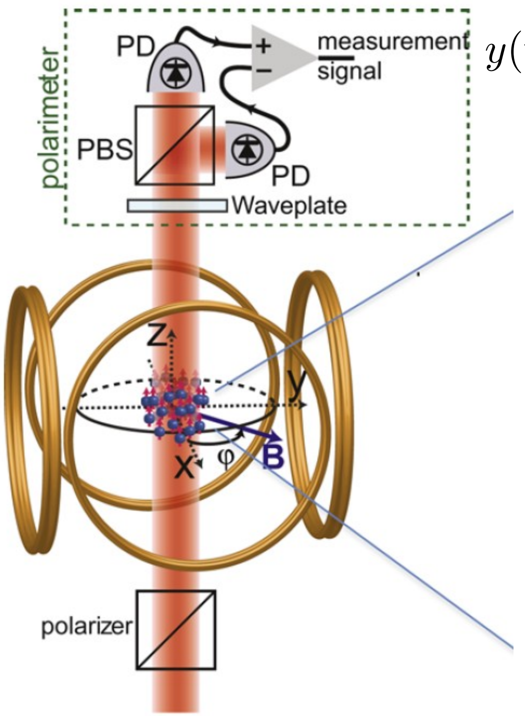
B) Sensing is performed **continuously in time**.

"*Noisy atomic magnetometry in real time*"
[**arXiv:2103.12025**]

Julia Amoros Binefa → Poster Session A

Noisy atomic magnetometry in real time

<u>Measurement</u>: **light probing** of the total angular momentum by the **Faraday effect**.



$y(t) \propto \Theta_{\rm FR}$

Polarisation of the probe light rotated by a **Faraday angle:**

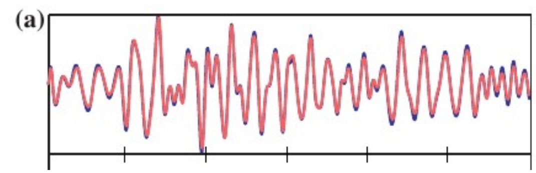
$$\Theta_{\rm FR} \;=\; g\,\hat{J}_z + \ldots$$

(weak interaction: **<u>off-resonance</u>**, <u>linear</u> interaction)

For <u>**quantum effects**</u>**:** operation beyond the **shot-noise** (limit)

$$y(t) = \tilde{g}\left\langle \hat{J}_z \right\rangle + \sigma W_t$$

white Gaussian noise
(**Wiener** stochastic process)



Real-time operation: need for <u>**Bayesian estimation**</u> of the **fluctuating** magnetic field:

$$\Delta^2 \tilde{B}_t = \mathbb{E}_{p(B_t,\boldsymbol{y}_{\leq t})}\left[(B_t - \tilde{B}_t(\boldsymbol{y}_{\leq t}))^2\right] = \int dB_t\; p(B_t)\; \mathbb{E}_{p(\boldsymbol{y}_{\leq t}|B_t)}\left[(B_t - \tilde{B}_t(\boldsymbol{y}_{\leq t}))^2\right]$$

prior distribution describing the field at time *t*: $p(B_t) = \int dB_0\; p(B_t|B_0)\; p(B_0)$

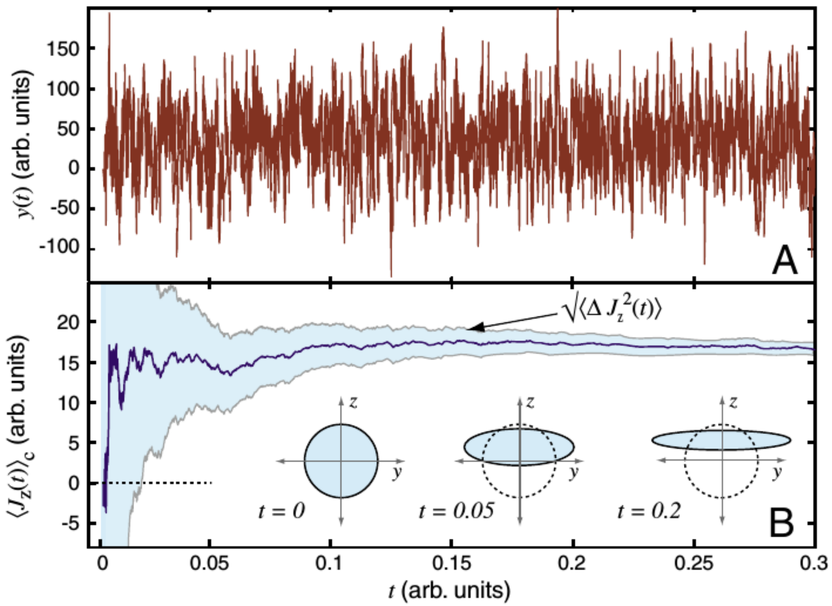
[arXiv:2103.12025]

Noisy atomic magnetometry in real time

VOLUME 91, NUMBER 25 PHYSICAL REVIEW LETTERS week ending 19 DECEMBER 2003

Quantum Kalman Filtering and the Heisenberg Limit in Atomic Magnetometry

JM Geremia,* John K. Stockton, Andrew C. Doherty, and Hideo Mabuchi
Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena, California, 91125, USA
(Received 27 June 2003; published 19 December 2003)



Quantum continuous measurement theory:

Measurement dynamics:

$$y(t)dt = 2\eta\sqrt{M}\langle\hat{J}_z(t)\rangle_c dt + \sqrt{\eta}\,dW(t),$$

Ensemble dynamics:

$$d\hat{\rho}_c(t) = -i\gamma B[\hat{J}_y, \hat{\rho}_c]dt + M\mathcal{D}[\hat{J}_z]\hat{\rho}_c dt + \sqrt{M\eta}\,\mathcal{H}[\hat{J}_z]\hat{\rho}_c dW(t),$$

measurement-induced decoherence (noise)

measurement-induced non-linear (unitary, stochastic) evolution

(Linear-)Gaussian regime: $\hat{X}(t) = \frac{\hat{J}_y}{\sqrt{|\langle\hat{J}_x(t)|\rangle}}$ $\hat{P}(t) = \frac{\hat{J}_z}{\sqrt{|\langle\hat{J}_x(t)|\rangle}}$ such that $[\hat{X}(t), \hat{P}(t)] = i\frac{\hat{J}_x}{\left|\langle\hat{J}_x(t)|\right\rangle|} \approx i$

Heisenberg scaling of sensitivity can be achieved for a **constant** field:

$$\Delta^2\tilde{B} \propto \frac{1}{t}\cdot\frac{1}{N} \quad \longrightarrow \quad \Delta^2\tilde{B} \propto \frac{1}{t^3}\cdot\frac{1}{N^2}$$

at short times *t<<1/M*

[arXiv:2103.12025]

Noisy atomic magnetometry in real time

Questions:

1) Is this still true if one includes noise: ***collective decoherence*** of the ensemble and ***B-field fluctuations***?

2) If not, then maybe there exists a ***better continuous measurement*** **scheme** that still allows for Heisenberg scaling?

Answers:

1) **No**. There exists then a general lower bound on sensitivity imposed by noise:

$$\Delta^2 B_t \geq \frac{1}{\gamma_g^2} \frac{\gamma_y}{t}$$

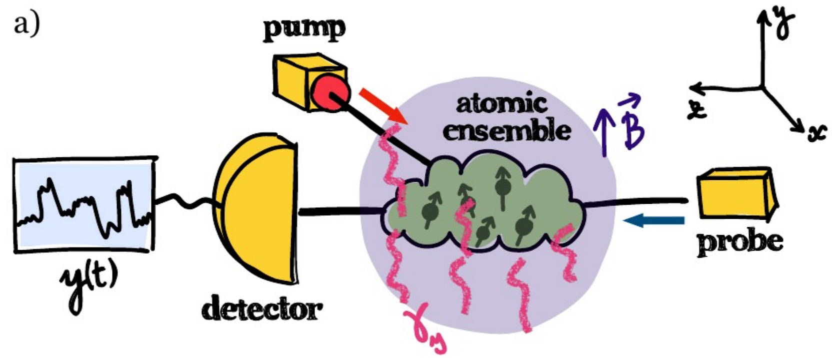
noise in the B-field direction

no *N*-dependence, sensitivity no longer increases with atom-number

2) The aforementioned measurement attains this bound within regimes of interest, so there is **no need to consider more elaborate detection schemes** (e.g. involving non-linearities etc).

[arXiv:2103.12025]

Noisy atomic magnetometry in real time



B-field fluctuations *(Orstein-Uhlenbeck process):* $$dB_t = -\chi B_t\, dt + dW_B,$$

Ensemble dynamics:

$$d\rho_{(c)}(t) = -\, i\gamma_g B_t[\hat{J}_y, \rho_{(c)}(t)]\, dt + \boxed{\sum_{\alpha=x,y,z} \gamma_\alpha \mathcal{D}[\hat{J}_\alpha]\rho_{(c)}(t)\, dt}$$
$$+ M\mathcal{D}[\hat{J}_z]\rho_{(c)}(t)\, dt + \sqrt{M\eta}\mathcal{H}[\hat{J}_z]\rho_{(c)}(t)\, dW,$$

global decoherence

J_z-component dynamics in the linear-Gaussian regime:

$$d\langle \hat{J}_z(t)\rangle_{(c)} = -\gamma_g B_t J e^{-rt/2} dt + 2\sqrt{\eta M}\, \langle \Delta^2 J_Z(t)\rangle_{(c)}\, dW,$$
$$d\langle \Delta^2 \hat{J}_z(t)\rangle_{(c)} = -4M\eta\, \langle \Delta^2 J_Z(t)\rangle^2_{(c)}\, dt + \gamma_y J^2 e^{-rt} dt,$$

Measurement dynamics:

$$y(t)dt = 2\eta\sqrt{M}\, \langle \hat{J}_z(t)\rangle_{(c)}\, dt + \sqrt{\eta}\, dW.$$

Optimal estimator of the B-field as the ***Kalman Filter****:*

$$\frac{\mathrm{d}\tilde{B}_t}{\mathrm{d}t} = \mathbf{F}_t \tilde{B}_t + \mathbf{K}_t \left(y(t) - \mathbf{H}_t \tilde{B}_t \right)$$

[arXiv:2103.12025]

Noisy atomic magnetometry in real time

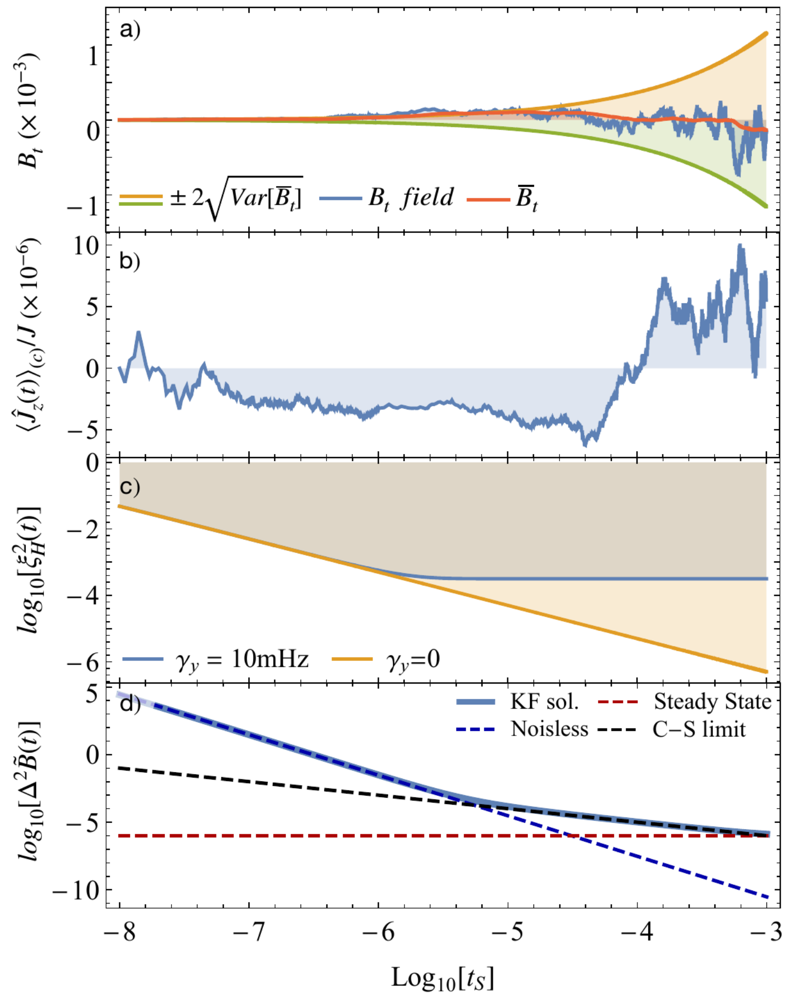
Dynamics of sensor in real time:

B-field fluctuations:
(small, to preserve linear-Gaussian approximation)

Measured signal:

(Conditional) spin-squeezing:

<u>Sensitivity:</u>



Classical-Simulation (C-S) limit
based on the
Bayesian Cramer-Rao Bound

$$\Delta^2 \tilde{B}_t \ \geq \sqrt{\frac{\gamma_y q_B}{\gamma^2}} \coth\left(t\sqrt{\frac{q_B \gamma^2}{\gamma_y}}\right)$$

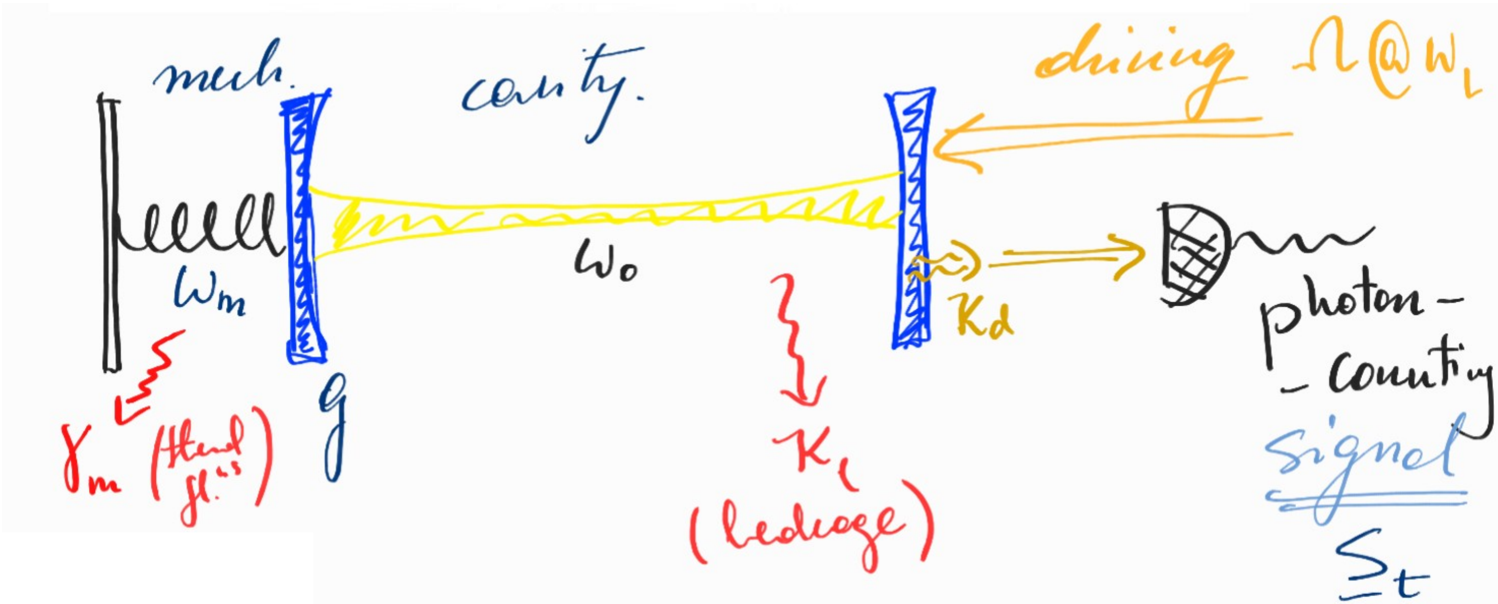
Quantum sensors: three challenges

C) **Efficient statistical inference from the recorded data is crucial.**

"*Enhancing performance of optomechanical sensors by continuous photon-counting*"

Lewis Clark & Bartosz Markowicz → Poster Sessions A&B

Optomechanics with photon-counting



Dynamics in the rotating frame of the driving laser field **beyond linear regime**: $\Delta = \omega_{\mathrm{L}} - \omega_0$

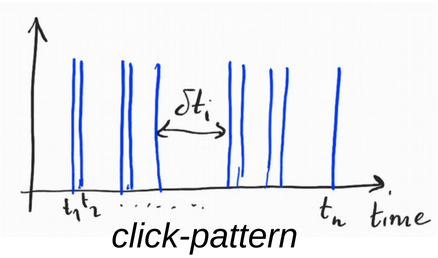
$$H = -\hbar\Delta a^\dagger a + \hbar\omega_{\mathrm{M}} b^\dagger b + \hbar g a^\dagger a (b + b^\dagger) + \frac{1}{2}\left(\Omega a + \Omega^* a^\dagger\right)$$

Dissipation and **photon-counting as a continuous measurement**:

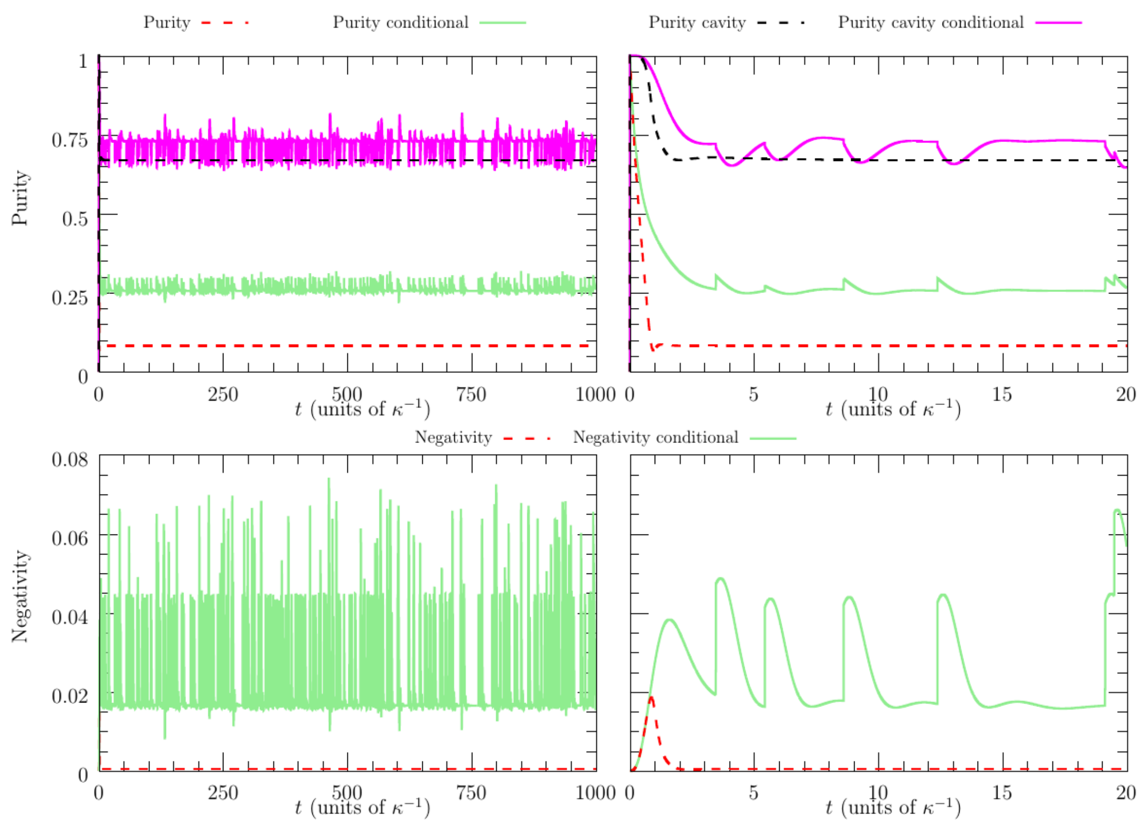
$$\begin{aligned}
\mathrm{d}\rho = &-\frac{\mathrm{i}}{\hbar}[H,\rho]\,\mathrm{d}t + \left(\kappa_{\mathrm{l}} a\rho a^\dagger - \frac{\kappa}{2}\left[a^\dagger a, \rho\right]_+ + \underline{\kappa_{\mathrm{d}} \mathrm{Tr}\left(a^\dagger a \rho\right)\rho}\right)\mathrm{d}t \\
&+ \gamma(\bar{m}+1)\left(b\rho b^\dagger - \frac{1}{2}\left[b^\dagger b, \rho\right]_+\right)\mathrm{d}t + \gamma\bar{m}\left(b^\dagger \rho b - \frac{1}{2}\left[b b^\dagger, b\right]_+\right)\mathrm{d}t \\
&+ \left(\frac{a\rho a^\dagger}{\mathrm{Tr}\left(a^\dagger a \rho\right)} - \rho\right)\underline{\mathrm{d}N_t^{(\kappa_{\mathrm{d}})}}.
\end{aligned}$$

Optomechanics with photon-counting

$$
\begin{aligned}
\mathrm{d}\rho = &-\frac{\mathrm{i}}{\hbar}\left[H,\rho\right]\mathrm{d}t + \left(\kappa_1 a\rho a^\dagger - \frac{\kappa}{2}\left[a^\dagger a,\rho\right]_+ + \kappa_\mathrm{d}\mathrm{Tr}\left(a^\dagger a\rho\right)\rho\right)\mathrm{d}t \\
&+ \gamma\left(\bar{m}+1\right)\left(b\rho b^\dagger - \frac{1}{2}\left[b^\dagger b,\rho\right]_+\right)\mathrm{d}t + \gamma\bar{m}\left(b^\dagger\rho b - \frac{1}{2}\left[bb^\dagger, b\right]_+\right)\mathrm{d}t \\
&+ \left(\frac{a\rho a^\dagger}{\mathrm{Tr}\left(a^\dagger a\rho\right)} - \rho\right)\underline{\mathrm{d}N_t^{(\kappa_\mathrm{d})}}\,.
\end{aligned}
$$

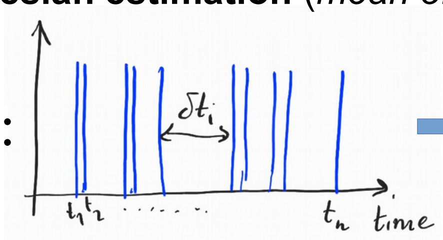


Continuous measurement affects ***entanglement*** **(two-stage architecture)** between cavity&mechanics d.o.f.s:



Optomechanics with photon-counting

Optimal Bayesian estimation (*mean of the posterior*) of parameters from a given “click-pattern” (*D*):

$D:$ 

click-pattern

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)} = \frac{P(D|\theta)P(\theta)}{\int \mathrm{d}\theta P(D|\theta)P(\theta)}$$

$$\tilde{\theta}(D) = \int \mathrm{d}\theta\, \theta\, P(\theta|D)$$

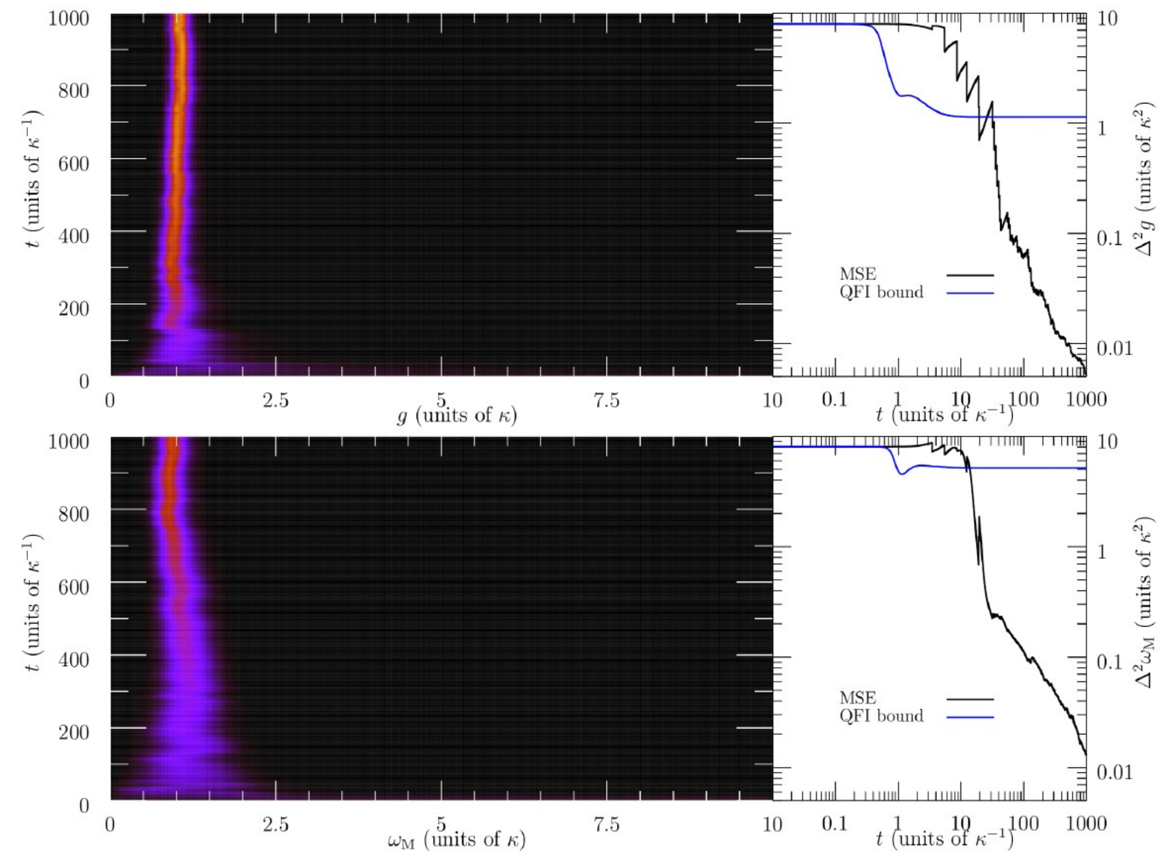
Estimation of:

coupling strength

$(\theta = g)$

mechanical frequency

$(\theta = \omega_M)$



Quickly outrun the best possible single-shot measurement scenario!

Conclusions

1) ***Quantum sensors*** are built based on a ***two-stage architecture***, so in order to carefully describe their operation one should move away from the idealistic approaches to quantum metrology.

2) Still, one can systematically incorporate the ***impact of measurement imperfections*** (read-out noise) into the abstract frameworks.
"*Quantum metrology with imperfect measurements*" [arXiv:2109.0116]

3) In order to deal with ***quantum sensors operating in 'real time'***, one must resort to frameworks of *continuous measurements* and *Bayesian estimation theory*.

4) Then, in ***presence of decoherence (and field fluctuations)*** one can derive *ultimate bounds on attainable sensitivity* that are determined solely by the noise.
"*Noisy atomic magnetometry in real time*" [arXiv:2103.12025]

5) Bayesian inference can be efficiently used (at some computational cost...) to infer parameters of ***quantum optomechanical sensors*** operating in *real time* also within the *non-linear regime*.